

# Stable Large-Scale Perturbations In Interacting Dark-Energy Model

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## Abstract

It is found that the evolutions of density perturbations on the super-Hubble scales are unstable in the model with dark-sector interaction  $Q$  proportional to the energy density of cold dark matter (CDM)  $\rho_m$  and constant equation of state parameter of dark energy  $w_d$ . In this paper, to avoid the instabilities, we suggest a new covariant model for the energy-momentum transfer between DE and CDM. Then we show that the large-scale instabilities of curvature perturbations can be avoided in our model in the universe filled only by DE and CDM. Furthermore, by including the additional components of radiation and baryons, we calculate the dominant non-adiabatic modes in the radiation era and find that the modes grow in the power law with exponent at the order of unit.

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**Key words:** dark energy, dark matter, interaction

## 1 Introduction

We are convinced by the increasing observations [1, 2, 3] that the present universe is dominated by the two components: dark energy (DE) [4, 5] and cold dark matter (CDM). Currently, the two components are only indirectly detected via the total gravitational effects. And then this produces an degeneracy: the two dark components might interact mutually without violating the observational constraints [6, 7]. And furthermore it is found that an appropriate interaction can help to alleviate the coincidence problem [8, 9],

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namely why DE and CDM are comparable in size exactly today [5]. Different interacting models of dark energy have been investigated intensively [10, 11].

Generally, a model with interaction between DE and CDM is described in the background by the two continuity equations

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (1)$$

$$\dot{\rho}_d + 3H(1 + w_d)\rho_d = -Q, \quad (2)$$

where  $Q$  denotes the phenomenological interaction term between DE and CDM;  $\rho_m$  and  $\rho_d$  are the energy densities of CDM and DE respectively;  $w_d \equiv p_d/\rho_d$  is the equation of state parameter of DE;  $p_d$  is the pressure density of DE;  $H \equiv \dot{a}/a$  is the Hubble parameter;  $a(t)$  is the scale factor in the Friedmann-Robertson-Walker (FRW) metric; a dot denotes the derivative with respect to the cosmic time  $t$ . In the note we do not allow the phantom case  $w_d < -1$ . Owing to the lack of the knowledge of micro-origin of the interaction, usually the interaction term is parameterized in a simple form as [8]

$$Q = 3H(\alpha\rho_m + \beta\rho_d), \quad (3)$$

where  $\alpha$  and  $\beta$  are positive constants. The interaction term  $Q$  would influence not only the background dynamics of the universe, but also the growth of the perturbations of the cosmological fluids [12, 13, 14].

Recently, in Ref.[15], by modeling DE as a fluid with constant  $w_d > -1$ , the authors investigated the evolution of the linear density perturbations and found that the combination of constant  $w_d$  and the simple interaction form  $Q$  given in Eq.(3) leads to an instability: the curvature perturbation on the super-Hubble scales blows up in the early universe [15]. The explicit models investigated in Ref.[15] included the two cases of  $\beta = 0$  and  $\alpha = \beta$  in Eq.(3). Further more, in [17], it is concluded that the perturbations in the dark energy become unstable for any model with constant  $w_d > -1$  and non-zero  $\alpha$ , no matter how small the parameter  $\alpha$  is made. In [16, 17, 18], the case of  $\alpha = 0$  was surveyed and it was found that the instability can be avoided if  $\beta$  is made small enough. In [19], by modeling DE as a quintessence field, the author found that the instability can also be avoided even for the interaction  $Q$  proportional to  $\rho_m$ .

Then it seems that a model with constant  $w_d$  and non-zero  $\alpha$  in Eq.(3) would be ruled out as a viable interacting model. To cure the curvature perturbations, the authors in Ref.[20] suggested a covariant model for energy-momentum transfer between DE and CDM, and showed that the instabilities of the curvature perturbation on the large scales can be avoided in the covariant model with constant  $w_d$  and  $Q = 3\alpha H\rho_m$ . In Ref.[20], the authors defined the effective EMTs of CDM and DE respectively as

$$T_{\text{em}}^{\mu\nu} = \rho_m u_m^\mu u_m^\nu + p_m^{\text{eff}}(u_m^\mu u_m^\nu + g^{\mu\nu}), \quad (4)$$

$$T_{\text{ed}}^{\mu\nu} = \rho_d u_d^\mu u_d^\nu + p_d^{\text{eff}}(u_d^\mu u_d^\nu + g^{\mu\nu}), \quad (5)$$

where  $p_m^{\text{eff}} \equiv -\alpha\rho_m$ ,  $p_d^{\text{eff}} \equiv p_d + \alpha\rho_m$ , and  $u_m^\mu$  and  $u_d^\mu$  are the four velocities of CDM and DE respectively. The two effective EMTs are taken to be conserved [20]

$$T_{\text{em};\nu}^{\mu\nu} = T_{\text{ed};\nu}^{\mu\nu} = 0, \quad (6)$$

and the Einstein equation are assumed to be [20]

$$G^{\mu\nu} = 8\pi G(T_{\text{em}}^{\mu\nu} + T_{\text{ed}}^{\mu\nu}). \quad (7)$$

Based on Eqs.(6) and (7), the evolution of the curvature perturbations were surveyed in [20] and no instabilities were found. The model defined in Eqs.(6) and (7) has a distinguishable feature that the total EMT of DE and CDM is not conserved at the perturbative level  $(T_m^{\mu\nu} + T_d^{\mu\nu})_{;\nu} \neq 0$ , while, in other covariant models, the total EMT is always taken to be conserved in order to match the Einstein tensor. Of course, in [20], the non-conserved total EMT does not cause problems since it is the total effective EMT that appears on the right-hand side of the Einstein equation and is guaranteed to be conserved by Eq.(6).

But it is still puzzling and discomforting that the total EMT of DE and CDM is not conserved. We try to solve the puzzlement by suggesting a new covariant model in this paper. We consider DE as a fluid with constant  $w_d$  that is coupled to CDM via a covariant energy-momentum transfer, from which  $Q = 3\alpha H\rho_m$  can be reduced at the background level. We show that the total EMT of CDM and DE is conserved and the stabilities of the curvature perturbation on the large scales can be avoided in our new covariant model.

This paper is organized as follows. In Sec.2, we display our new covariant model for the energy-momentum transfer in the dark sector. In Sec.3, by assuming the universe filled only by DE and CDM, we survey the evolution of the density perturbations and show that instabilities on the large scales can be avoided. In Sec.4, by considering the effects of the radiation (photons and neutrinos) and baryons, we investigate the dominant non-adiabatic mode in the radiation era and show that no non-adiabatic modes blow up. Finally, conclusions and discussions are given.

## 2 New Covariant Model for Interaction in Dark Sector

Usually, the covariant form for energy-momentum transfer is taken to be [15, 21]

$$\nabla_\nu T_A^{\mu\nu} = Q_A^\mu, \quad (8)$$

where  $A = m, d$  to denote CDM and DE respectively, and the condition  $\sum_A Q_A^\mu = 0$  is imposed in order for the total energy-momentum tensor to be conserved. By comparing

Eqs.(6) and (8), we can get the covariant interacting terms of the model in Eq.(6) as [20]

$$Q_m^\mu = \alpha[\rho_m(u_m^\mu u_m^\nu + g^{\mu\nu})]_{;\nu}, \quad (9)$$

$$Q_d^\mu = -\alpha[\rho_m(u_d^\mu u_d^\nu + g^{\mu\nu})]_{;\nu}. \quad (10)$$

Clearly, at the perturbation level,  $Q_m^\mu \neq -Q_d^\mu$  and so the total EMT of DE and CDM is not conserved.

Yet, motivated by Eqs.(9) and (10), we find that a new covariant model can be constructed as

$$T_m^{\mu\nu}{}_{;\nu} = (\rho_m u_m^\mu u_m^\nu)_{;\nu} = -Q_d^\mu, \quad (11)$$

$$T_d^{\mu\nu}{}_{;\nu} = [\rho_d u_d^\mu u_d^\nu + p_d(u_d^\mu u_d^\nu + g^{\mu\nu})]_{;\nu} = Q_d^\mu, \quad (12)$$

where  $Q_d^\mu$  is given in Eq.(10). Obviously, in this model the total EMT of DE and CDM is conserved. Indeed, this new model is just a usual type of Eq.(8) with the choice of the covariant energy-momentum transfer as

$$Q_m^\mu = -Q_d^\mu = \alpha[\rho_m(u_d^\mu u_d^\nu + g^{\mu\nu})]_{;\nu}.$$

Our model is similar to the case in [22, 23, 24] where

$$Q_m^\mu = -Q_d^\mu = \beta(\phi)T_{m\nu}^\nu \nabla^\mu \phi,$$

in the sense that the interaction is determined by the energy density of CDM and the four velocity of DE. It can be easily checked that Eqs.(1) and (2) with  $Q = 3\alpha H\rho_m$  can be deduced from Eqs.(11) and (12) at the background level, respectively. Notably, this model has a similar feature to the one in Ref.[20] that the global quantity  $H$  in  $Q = 3\alpha H\rho_m$  is explained to be a local quantity  $u_{d;\nu}^\nu$ .

Now together with the Einstein equation

$$G^{\mu\nu} = 8\pi G(T_m^{\mu\nu} + T_d^{\mu\nu}), \quad (13)$$

we can study the evolution of the curvature perturbation on the large scales to check whether the large-scale instabilities could be avoided.

### 3 Evolution of Density Perturbations

In this section, we apply the new model suggested in the last section to survey the evolution of the density perturbations by modeling DE as a fluid with constant  $w_d$ . For simplicity, we consider a flat universe filled only by DE and CDM. We choose the conformal Newtonian gauge and then the perturbed FRW metric in the conformal is given by

$$ds^2 = a^2(\tau)[-(1 + 2\phi)d\tau^2 + (1 - 2\psi)d\mathbf{x}^2], \quad (14)$$

where  $\phi$  and  $\psi$  denote the scalar perturbations. The Friedmann equation reads

$$\mathcal{H}^2 \equiv \left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_d) \quad (15)$$

Hereafter, primes denote the derivatives respect to  $\tau$ . With  $Q = 3\alpha H \rho_m$  and constant  $w_d$ , we can obtain the background evolutions of  $\rho_m$  and  $\rho_d$  from Eqs.(1) and (2) as

$$\rho_m = \rho_{m0} a^{-3(1-\alpha)}, \quad (16)$$

$$\rho_d = \rho_{d0} a^{-3(1+w_d)} + \left(\frac{\alpha}{\alpha + w_d}\right) \rho_{m0} a^{-3} (a^{-3w_d} - a^{3\alpha}). \quad (17)$$

In this paper, we use the subscript 0 to denote the present value of the corresponding parameter and  $a_0 = 1$ .

### 3.1 Evolving equations of Perturbations

When the perturbed metric in Eq.(14) is considered, the four velocities of CDM and DE are

$$u_m^\mu = a^{-1} (1 - \phi, \partial_i v_m), \quad u_d^\mu = a^{-1} (1 - \phi, \partial_i v_d), \quad (18)$$

where  $v_m$  and  $v_d$  are the peculiar velocity potentials of CDM and DE respectively. Usually, we define the volume expansion rates of CDM and DE (in Fourier space) respectively as

$$\theta_m = -k^2 v_m, \quad \theta_d = -k^2 v_d. \quad (19)$$

We use  $\delta\rho_m$ ,  $\delta\rho_d$  and  $\delta p_d$  to denote the first-order perturbations of the corresponding parameters, and introduce two dimensionless first-order parameters as

$$\delta_m = \frac{\delta\rho_m}{\rho_m}, \quad \delta_d = \frac{\delta\rho_d}{\rho_d}. \quad (20)$$

The total curvature perturbation on the constant- $\rho$  ( $\rho = \rho_m + \rho_d$ ) surface is defined as

$$\zeta = -\psi - \mathcal{H} \frac{\delta\rho_m + \delta\rho_d}{\rho'_m + \rho'_d}. \quad (21)$$

Here, following the analysis in Ref.[15], we take

$$\delta p_d = \delta\rho_d + (1 - w_d) [3\mathcal{H}(1 + w_d + \alpha \frac{\rho_m}{\rho_d}) \rho_d] \frac{\theta_d}{k^2}. \quad (22)$$

Then from Eq.(16), we get two equations as

$$\delta'_m - 3(1 - \alpha)\psi' + \theta_m - \alpha\theta_d = 0, \quad (23)$$

$$\theta'_m + \mathcal{H}(1 + 3\alpha)\theta_m - k^2(1 - \alpha)\phi = \alpha [\theta'_d + \mathcal{H}(1 + 3\alpha)\theta_d - k^2\delta_m]. \quad (24)$$

And from Eq.(17), we get other two equations as

$$\begin{aligned} \delta'_d + 3\mathcal{H}(1 - w_d - \alpha \frac{\rho_m}{\rho_d})\delta_d + 9\mathcal{H}^2(1 - w_d)(1 + w_d + \alpha \frac{\rho_m}{\rho_d})\frac{\theta_d}{k^2} \\ + 3\alpha\mathcal{H}\frac{\rho_m}{\rho_d}\delta_m + (1 + w_d + \alpha \frac{\rho_m}{\rho_d})(\theta_d - 3\psi') = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} \theta'_d - 2\mathcal{H}\theta_d - 3\mathcal{H}\frac{\alpha(1 - \alpha)\frac{\rho_m}{\rho_d}}{1 + w_d + \alpha \frac{\rho_m}{\rho_d}}\theta_d - k^2\phi \\ - k^2\frac{\alpha \frac{\rho_m}{\rho_d}}{1 + w_d + \alpha \frac{\rho_m}{\rho_d}}\delta_m - k^2\frac{\delta_d}{1 + w_d + \alpha \frac{\rho_m}{\rho_d}} = 0. \end{aligned} \quad (26)$$

In the conformal Newtonian gauge, the first-order Einstein equations (13) gives us [25]

$$3\mathcal{H}\psi' + k^2\psi + 3\mathcal{H}^2\phi = -4\pi G a^2(\delta_m\rho_m + \delta_d\rho_d), \quad (27)$$

$$k^2\psi' + k^2\mathcal{H}\phi = 4\pi G a^2[\rho_m\theta_m + (1 + w_d)\rho_d\theta_d] \quad (28)$$

$$\psi'' + \mathcal{H}(2\psi' + \phi') + (2\frac{a''}{a} - \mathcal{H}^2)\phi + \frac{k^2}{3}(\psi - \phi) = 4\pi G a^2\delta p_d \quad (29)$$

$$\psi - \phi = 0. \quad (30)$$

Only two of the above equations are independent. Choosing any two of them and using Eqs.(23)-(26), we can solve these evolving equations numerically if the initial conditions are given.

### 3.2 Adiabatic Initial Conditions

In the early universe,  $a \ll 1$ , Eqs.(16) and (17) indicate

$$\frac{\rho_m}{\rho_d} \rightarrow -\frac{w_d + \alpha}{\alpha}, \quad (31)$$

and then, from Eq.(15), we have

$$\mathcal{H} = \frac{2}{1 - 3\alpha}\tau^{-1}, \quad \tau = \frac{2}{(1 - 3\alpha)H_0}\sqrt{\frac{w_d + \alpha}{w_d\Omega_{m0}}}a^{\frac{1}{2}(1 - 3\alpha)}, \quad (32)$$

where

$$\Omega_{m0} \equiv \frac{8\pi G\rho_{m0}}{3H_0^2}.$$

Here we adopt the adiabatic initial conditions to study the evolution of the density perturbations on the super-Hubble scales ( $k \ll aH$ ). To the lowest order in  $k\tau$ , we can

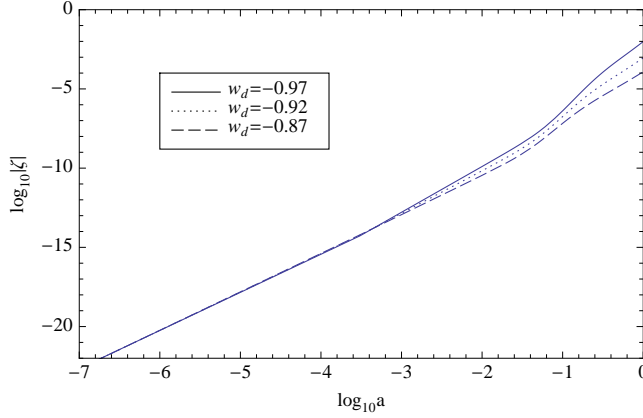


Fig. 1:  $\log_{10} |\zeta|$  versus  $\log_{10} a$  in the interacting model for fixed  $\Omega_{m0} = 0.3$ ,  $h = 0.67$ ,  $k = 1.5 \times 10^{-4} \text{Mpc}^{-1}$ ,  $\alpha = 10^{-3}$  and different  $w_d$ .

set the adiabatic conditions as

$$\phi = \psi = A_\phi = \text{Const.}, \quad (33)$$

$$\delta_m = \delta_d = -2A_\phi, \quad (34)$$

$$\theta_m = \theta_d = \frac{1-3\alpha}{3(1-\alpha)} k^2 \tau A_\phi. \quad (35)$$

### 3.3 Evolution of Curvature Perturbation on Large Scales

Now using the initial conditions in the last subsection, we can obtain the evolution of the scalar perturbations by solving Eqs.(23)-(27) and (30) numerically, and then get the evolution of the curvature perturbation  $\zeta$  defined in Eq.(21). We display the results in Fig.1 and Fig.2. Here we have taken  $A_\phi = 10^{-25}$  and  $a_0 = 1$ , and fixed the values of some parameters:  $\Omega_{m0} = 0.3$ ,  $k = 1.5 \times 10^{-4} \text{Mpc}^{-1}$ ,  $H_0 = 100h \text{ km sec}^{-1} \text{Mpc}^{-1}$  and  $h = 0.67$ . The evolving curves in Fig.1 and Fig.2 manifest the regular growth in power law and no instabilities occur.

## 4 Dominant Non-adiabatic Modes

In the last section, by solving the evolving equations numerically with the adiabatic initial conditions, we show that instabilities of the curvature perturbation on the large scales are removed in our new covariant model. But the conclusion is obtained in the universe filled only by CDM and DE. In this section, we consider the effects of baryons, photons and neutrinos by calculating the dominant non-adiabatic modes deep in the radiation era. If the dominant non-adiabatic modes do not grow rapidly, we believe that

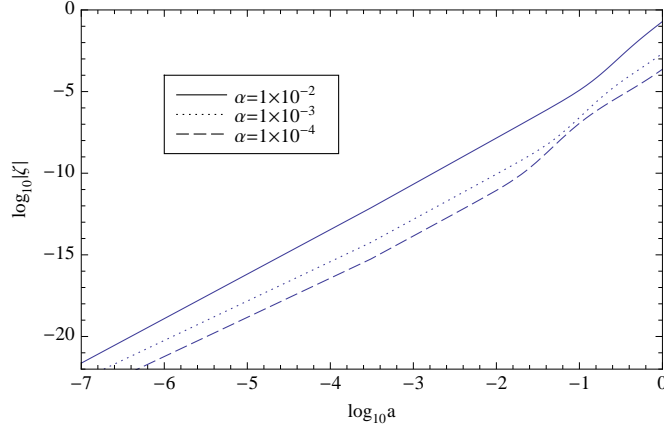


Fig. 2:  $\log_{10} |\zeta|$  versus  $\log_{10} a$  in the interacting model for fixed  $\Omega_{m0} = 0.3$ ,  $h = 0.67$ ,  $k = 1.5 \times 10^{-4} \text{Mpc}^{-1}$ ,  $w_d = -0.94$  and different  $\alpha$ .

the instabilities can be avoided even when radiations and baryons are involved in the universe.

The  $A$ -fluid EMT including perturbations is taken to be

$$T_A^{\mu\nu} = (\rho_A + p_A)u_A^\mu u_A^\nu + p_A g^{\mu\nu} + \pi_A^{\mu\nu}, \quad (36)$$

where  $u_A^\mu$  is the four velocity

$$u_A^\mu = a^{-1} \left( 1 - \phi, \partial_i v_A \right).$$

We have allowed an anisotropic shear perturbation  $\pi_A^{\mu\nu}$ , and  $A = m, d, b, \gamma, \nu$  to denote the corresponding parameters of CDM, DE, baryons, photons and neutrinos. Following Ref.[15], we take  $\pi_m^{\mu\nu} = \pi_d^{\mu\nu} = \pi_b^{\mu\nu} = \pi_\gamma^{\mu\nu} = 0$  and

$$\pi_\nu^{0\mu} = 0, \quad \pi_\nu^{ij} = a^{-2} \left( \partial_i \partial_j - \frac{1}{3} \delta^{ij} \right) \pi_\nu. \quad (37)$$

For DE and CDM, the continuity equations (11) and (12) still hold.

Early in the radiation era, the Friedmann equation reads

$$\mathcal{H}^2 a^{-2} = \frac{8\pi G}{3} (\rho_\gamma + \rho_\nu) = \frac{8\pi G}{3} \rho_{r0} a^{-4}. \quad (38)$$

Here we use the subscript  $r$  to denote the corresponding parameter of radiation which consists of photons and neutrinos. Then we have

$$a = \sqrt{\Omega_{r0}} H_0 \tau, \quad \mathcal{H} = \tau^{-1}, \quad \Omega_{r0} \equiv \frac{8\pi G \rho_{r0}}{3H_0^2}. \quad (39)$$



In the radiation era, the perturbed Einstein equations give us that

$$3\tau^{-1}\psi' + k^2\psi + 3\tau^{-2}\phi = -4\pi Ga^2\left(\delta\rho_m + \delta\rho_d + \sum_A \delta\rho_A\right), \quad (40)$$

$$k^2(\psi' + \tau^{-1}\phi) = 4\pi Ga^2\left[\rho_m\theta_m + (1 + w_d)\rho_d\theta_d + \sum_A (\rho_A + p_A)\theta_A\right], \quad (41)$$

$$\psi'' + 2\tau^{-1}\psi' + \tau^{-1}\phi' - \tau^{-2}\phi + \frac{k^2}{3}(\psi - \phi) = 4\pi Ga^2\left(\delta p_d + \sum_A \delta p_A\right), \quad (42)$$

$$\psi - \phi = 8\pi G\pi_\nu, \quad (43)$$

where  $A$  runs over  $b, \gamma$ , and  $\nu$ .

For CDM and DE, the perturbed continuity equations are given by Eqs.(23)-(26). For baryons, the perturbed continuity equations (in Fourier space) are [15]

$$\delta'_b = -\theta_b + 3\psi', \quad (44)$$

$$\theta'_b = -\mathcal{H}\theta_b + k^2\phi, \quad (45)$$

and for photons [15]

$$\delta'_\gamma = -\frac{4}{3}\theta_\gamma + 4\psi', \quad (46)$$

$$\theta'_\gamma = \frac{1}{4}k^2\delta_\gamma + k^2\phi, \quad (47)$$

and for neutrinos [15]

$$\delta'_\nu = -\frac{4}{3}\theta_\nu + 4\psi', \quad (48)$$

$$\theta'_\nu = \frac{1}{4}k^2\delta_\nu + k^2\phi - k^2\sigma_\nu, \quad (49)$$

$$\sigma'_\nu = \frac{4}{15}\theta_\nu, \quad (50)$$

where  $\theta_A \equiv -k^2 v_A$  for  $A = b, \gamma, \nu$ , and  $\sigma_\nu \equiv 2k^2\pi_\nu/[3a^2(\rho_\nu + p_\nu)]$

In order to find the dominant non-adiabatic modes, we assume a leading-order power law for perturbations [15]

$$\psi = A_\psi(k\tau)^{n_\psi}, \quad \phi = A_\phi(k\tau)^{n_\phi}, \quad \delta_A = B_A(k\tau)^{n_A}, \quad \theta_A = C_A(k\tau)^{s_A}, \quad \sigma_\nu = D_\nu(k\tau)^{n_\sigma}. \quad (51)$$

Here the subscript  $A = m, d, b, \gamma$ , and  $\nu$  denotes the corresponding parameter of CDM, DE, baryons, photons and neutrinos, respectively. To the leading order in  $k\tau$ , the equa-

tions (23)-(26) and (40)-(50) may be solved, in terms of  $\psi$ :

$$\phi = J\psi, \quad J = 1 - \frac{16R_\nu}{5(n_\psi + 2)(n_\psi + 1) + 8R_\nu}, \quad (52)$$

$$\delta_\gamma = \delta_\nu = 4\psi, \quad \delta_b = 3\psi, \quad (53)$$

$$\theta_\gamma = \theta_\nu = \theta_b = \frac{J+1}{n_\psi + 1} k^2 \tau \psi, \quad \sigma_\nu = \frac{4}{15} \frac{\tau \theta_\nu}{n_\psi + 2} \quad (54)$$

$$\delta_m = 3(1 - \alpha)\psi, \quad (55)$$

$$\delta_d = \frac{2\Omega_{\tau 0}^{(1-3\alpha)/2}}{\alpha\Omega_{m0}H_0^{1+3\alpha}}(w_d + \alpha)(n_\psi + J + 2)\frac{\psi}{\tau^{1+3\alpha}}, \quad (56)$$

$$\theta_d = -\frac{n_\psi + 2}{9(1 - w_d)(1 - \alpha)} k^2 \tau \delta_d, \quad \theta_m = \alpha\theta_d, \quad (57)$$

where  $R_\nu \equiv \rho_\nu/(\rho_\gamma + \rho_\nu)$  and

$$n_\psi = \frac{-3w_d \pm \sqrt{9w_d^2 + 12w_d - 20}}{2}, \quad (58)$$

From Eq.(56), we should require

$$\text{Re}[n_\psi] \geq 1 + 3\alpha.$$

in order for the modes to be well behaved as  $k\tau \rightarrow 0$ . For  $w_d \sim -1$ , this leads to

$$-\frac{3}{2}w_d \geq 1 + 3\alpha \Rightarrow \alpha \lesssim \frac{1}{6}.$$

Correspondingly, the total curvature perturbation  $\zeta$  is defined as

$$\zeta = -\psi - \mathcal{H} \frac{\sum_A \delta\rho_A}{\sum_A \rho'_A}$$

where  $A$  runs over  $m, d, \gamma, \nu$  and  $b$ . Then  $\zeta$  can be expressed in terms of  $\psi$  as

$$\zeta = -\frac{1}{2}(n_\psi + J + 2)\psi. \quad (59)$$

For  $w_d \sim -1$ ,  $n_\psi$  is a complex number and

$$\text{Re}[n_\psi] \sim \frac{3}{2}.$$

So the dominant non-adiabatic modes grows in a power law with exponent at the order of unit and no instabilities are present.

## 5 Conclusions and Discussions

It was found in [15] that the evolution of the density perturbations is unstable in the model with constant  $w_d$  and  $Q$  proportional to  $\rho_m$ . To cure it, a covariant model for energy-momentum transfer in dark sector was suggested in [20]. Yet, the model in [20] seems to be “anomalous”, since it has a puzzling and discomfoting feature that the total EMT of DE and CDM is not conserved. In order to remove the “anomaly”, in this paper, we have suggested a new covariant model in which the total EMT of DE and CDM is conserved. By choosing  $Q = 3\alpha H \rho_m$  and using the new model, we survey the evolution of density perturbations in a universe filled by DE and CDM, and show that the large-scale instabilities can be avoided. Furthermore, we calculate the dominant non-adiabatic modes in the radiation era by including the other components of radiation and baryons in the universe, and find that the dominant non-adiabatic modes grow in the power law with exponent at the order of unit. This makes us believe that the instabilities can also be avoided even in a universe filled by radiation, matter and DE.

Qualitatively, it is easy to understand why the instabilities can be avoided in our new model. In the model in [15], it was shown that the coupling term  $Q$  leading the instability-driving term in the perturbed continuity equation for  $\theta_d$  during the radiation era

$$\theta'_d \sim \frac{\alpha}{1+w_d} \frac{\rho_m}{\rho_d} \mathcal{H} \theta_d \simeq -\frac{w_d + \alpha}{1+w_d} \mathcal{H} \theta_d. \quad (60)$$

If  $w_d$  is close to  $-1$ , this term becomes very large and causes the rapid growth of  $\theta_d$  during the radiation era. Nevertheless, in the model in [20], it was shown that the corresponding driving term in the continuity equation for  $\theta_d$  during the radiation era becomes

$$\theta'_d \sim \frac{\alpha}{1+w_d + \alpha(\rho_m/\rho_d)} \frac{\rho_m}{\rho_d} \mathcal{H} \theta_d \simeq -\frac{w_d + \alpha}{1 - \alpha} \mathcal{H} \theta_d. \quad (61)$$

Clearly, this term does not cause instabilities even for  $w_d \sim -1$ . So it is believed that the instabilities can be avoided in the model in [20]. In fact, the continuity equation for  $\theta_d$  in our new model, Eq.(26), are same to the one in [20]. Then the analysis based on Eq.(61) also apply here. So we believe that the instabilities can be avoided in our new model, as in the model in [20].

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